1

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{n} = \sin\theta(\cos\phi\,\hat{x} + \sin\phi\,\hat{y}) + \cos\theta\,\hat{z}$$

$$\hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \hat{n}\cdot\hat{\vec{S}} = \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\phi} \\ \sin\theta\, e^{i\phi} & -\cos\theta \end{pmatrix}$$

$$\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\cancel{\chi \cdot S \chi_\pm} \quad \hat{n}\cdot\hat{\vec{S}}\ \underbrace{\cancel{\chi}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}}_{\chi'_\pm} = \pm\frac{1}{2}\hbar\underbrace{\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}}_{\chi'_\pm}$$

$$\frac{\cancel{\hbar}}{2}\begin{pmatrix} \cos\theta\, c_1 + \sin\theta\, e^{-i\phi} c_2 \\ \sin\theta\, e^{i\phi} c_1 - \cos\theta\, c_2 \end{pmatrix} = \pm\frac{\cancel{\hbar}}{2}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$\cos\theta\, c_1 + \sin\theta\, e^{-i\phi} c_2 = \pm c_1 \qquad \bar{c}_2 = e^{-i\phi} c_2$$

$$\sin\theta\, e^{i\phi} c_1 - \cos\theta\, c_2 = \pm c_2$$

$$\cancel{\cos\theta} \quad \cos\theta\, c_1 + \sin\theta\, \bar{c}_2 = \pm c_1$$

$$\cancel{+\sin\theta} \quad \sin\theta\, c_1 - \cos\theta\, \bar{c}_2 = \pm \bar{c}_2$$

$$\cancel{c_1 = \pm(c_1\cos\theta +}$$

$$\cancel{\cos\theta\, x_1} \qquad \cos^2\theta/2 = \frac{1+\cos\theta}{2}$$

$$(\cos\theta \mp 1)c_1 + \sin\theta\, \bar{c}_2 = 0$$

$$\sin\theta\, c_1 - (\cos\theta \pm 1)\bar{c}_2 = 0$$

$$c_1 = \frac{\cos\theta \pm 1}{\sin\theta}\bar{c}_2 = \frac{\cos\theta \pm 1}{\sin\theta} e^{-i\phi} c_2$$

$$\chi'_+ : c_1 = \frac{\cancel{\cos\theta}\, 1+\cos\theta}{\sin\theta} e^{-i\phi} c_2 = \frac{\cancel{2}\cos^{\cancel{2}}(\theta/2)}{\cancel{2}\sin\theta/2\,\cancel{\cos\theta/2}} e^{-i\phi} c_2 = \frac{\cos(\theta/2)}{\sin(\theta/2)} e^{-i\phi} c_2$$

$$c_1 = \cos(\theta/2)\, e^{-i\phi/2}$$

$$c_2 = \sin(\theta/2)\, e^{+i\phi/2}$$